# Mathematical Model For Determination the Economically Justified Distance for Reactive Power Transmission in Electric Power System

Luma. N. M. Tawfiq & Manar. I. Ismail

**Abstract**— This paper gives detailed analyses to help determine the increase in production costs due to the transmission of reactive power. We designed mathematical models to determine an economically justified transmission distance for VARS in electric power systems. Standard voltages and line parameters are used for the computations. This models used to compute the economically justified distance for VARS transmission from substation Haditha Dam substation to Qa'im. MATLAB computer programming is used to obtain the numerical results.

**Index Terms**— Mathematical model, reactive power transmission costs, VARS transmission.

◆

## 1 INTRODUCTION

Exact solution and simulation of various engineering problems, especially control engineering problems, depend on convenient mathematical models for elements and subsystems of the system considered. Process of the transformation of the system's behavior to mathematical basis is called "mathematical modeling" [1], [2], [3], [4].

The social structures and the industrial development of any country depend primarily upon low cost and uninterrupted supply of electrical energy, Mehta et al. [5]. The process of modernization, increase in productivity, agriculture and industry basically depend upon the adequate supply of electrical energy, Gupta [6]. Generation of electrical energy is the conversion of energy available in different forms in nature to electrical energy. The ever increasing use of electrical energy for industrial, domestic and commercial purposes necessitated the bulk production of electrical energy. This bulk production is achieved with the help of suitable power production stations which are generally referred to as electric power generating stations or electric power plants. A generating station usually employs a prime mover coupled with an alternator to produce electric power. Electrical energy is generated at power stations which are usually situated far away from load centers.

Hence an extensive network of conductors between the power stations and the consumers is required. This network of conductors may be divided into two main components, called the transmission system and the distribution system. The transmission system is to deliver bulk power from power stations to load centers and large industrial consumers while the distribution system is to deliver power from substations to various consumers.

Electrical energy must be transmitted and distributed to the point of use as soon as it is needed. Transmission lines and other materials are needed to achieve this purpose. Transmission lines are materials or media that are used to transmit electric energy and signals from one point to another, specifically from a source to a load. They can be regarded as a set of conductors being run from one place to another and supported on transmission towers. This involves connections between an electric generating plant and a substation which is several hundred kilometers away. The transmission and distribution stages are very important to electric power system because without these stages the generated power cannot get to the load centers not to talk of getting to the final consumers, Mehta et al. [5], Atandare [7] and Wadhwa [8].

## 2. MOTIVATION FOR THE STUDY

A lot of research work had been carried out by scientists and engineers on the generation of power, reliability of transmission systems and reduction of losses on transmission lines: Bamigbola et al [9], considered the characterization of optimal control model of electric power generating systems using two control variables, Aderinto[10] developed a mathematical model for electric power generating system using the optimal control approach with one control variable, Okafor et al [11] assessed the reliability of transmission systems in Nigeria by using the general reliability function and calculating the reliability indices for six 330KV transmission lines in Nigeria. Bagriyanik et al [12] used a fuzzy multi-objective optimization and genetic algorithm-based method to find optimum power system operating conditions. In addition to active power losses, series reactive power losses of transmission system are also considered as one of the multiple objectives. Onohaebi et al [13] considered the relationship of the effect of distance and loadings on power losses using the existing 28 bus, 330KV Nigerian transmission network as a case study in his empirical modeling of power losses as a function of line loadings and lengths in the Nigeria 330KV transmission

---

*Luma. N. M. Tawfiq and Manar. I. Ismail are from Department of Mathematics, College of Education for Pure Science - Ibn Al-Haitham, Baghdad University, Baghdad, Iraq.*

lines, to mention a few. The mathematical models for the determination of voltages and currents on lossy electric power transmission lines has not been work upon by any of these researchers hence the need for this work. In this paper, we present the mathematical models for the determination of voltage and energy on lossy electric power transmission lines.

## 3. ELECTRICAL ENERGY

Energy is a basic necessity for the economic development of a nation. There are different forms of energy, but the most important form is the electrical energy. A modern and civilized society is so much dependent on the use of electrical energy. Electrical energy is transmitted by means of transmission lines which deliver bulk power from generating stations to load centers. The industrial development of any nation depends majorly upon the reliability of its interconnected electric power system. Availability of electric energy has been the most powerful vehicle for facilitating economic, industrial and social developments of any nation. When an electric power is generated in sufficient quantities, it needs to be transmitted in bulk to load centers and then distributed to individual consumers in proper form and quality at the lowest possible ecological and economic price. This electric power is transmitted by means of transmission lines [14].

## 4. REACTIVE POWER

Is one of a class of power system reliability services collectively known as ancillary services, measured in volt-amperes reactive or VARs, ancillary services are essential for the reliable operation of the bulk power system. Reactive power flows when current leads or lags behind the voltage; typically, the current lags because of inductive loads like motors. Reactive power flow wastes energy and transmission capacity, and causes voltage droop. To correct this lagging power flow, leading reactive power (current leading voltage) is supplied to bring the current in phase with voltage [15]. Reactive power can be supplied from either static or dynamic VAR sources. Static sources are typically transmission and distribution equipment, such as static VAR compensators or capacitors at substations, and their cost has historically been included in the revenue requirement of the transmission owner (TO), and recovered through cost-of-service rates. By contrast, dynamic sources are typically energy producers, including generators capable of producing both real and reactive power, and synchronous condensers, which produce only reactive power [16].

## 4. REACTIVE POWER

Transmission of both active and reactive power lead to losses in the system as mentioned in the introduction. Since active power is usually generated specifically to compensate for load demand, it is the reactive power that is controlled to achieve a reduction of losses in the system.

When a power system is being designed and the parameters are yet to be determined, it is a generally accepted must to compensate for the predicted reactive power demand at the consuming end so as to reduce losses in the system. This reduction of the total transmitted power allows for the use of smaller conductors for transmission, leading to the reduction of system construction costs. Because of the expensive nature of the compensation equipment, the cost is also taken into account in determining the most economically justified distance for reactive power transmission..

## 6. ADDED INCREASE IN COST OF SYSTEMS EQUIPMENT

The total current in any system element of a three phase network is given as:

$$I = \frac{\sqrt{P^2 + q^2}}{\sqrt{3}\,V} = \frac{P\sqrt{1+\tan^2\theta}}{\sqrt{3}\,V} \qquad (1)$$

Where:
p = active power
q = reactive power

The cross section area of a power transmission conductor is given as,

$$F = \frac{I}{J} = \frac{p\sqrt{1+\tan^2\theta}}{\sqrt{3}\,VJ} \qquad (2)$$

where J is the current density of the conductor in A/mm2.

The total cost of transmission line per km due to the added losses is [17]:

$$B_L = (b_{0L}L + b_L FL) = (b_{0L} + b_L F) \qquad (3)$$

Where,
$b_L$: a variable constant reflecting increase in cost of conductor, \$/km.mm$^2$.
$b_{0L}$ : a fixed cost component of the conductor, \$/km.
L: total length of the conductor ,km.

Substitution of equation (2) into equation (3) gives equation (4),

$$B_L = \left(b_{0L} + \frac{b_L\,p\sqrt{1+\tan^2\theta}}{\sqrt{3}\,VJ}\right)L \qquad (4)$$

The equation of total cost without transmissionn reactive power (i.e., transmission purely active power, that is θ = 0) is:

$$B_{L2} = \left(b_{0L} + \frac{b_L\,p\sqrt{1+\tan^2 0}}{\sqrt{3}\,V\,J}\right)L \qquad (5)$$

The additional cost due to the transmission of apparent power as compared with the transmission of purely active power is expressed as:

$$\Delta B_L = \frac{b_L\,pL(\sqrt{1+\tan^2\theta} - 1)}{\sqrt{3}\,V\,J} \qquad (6)$$

The final equation, Increased cost per unit of reactive power transmission is given by:

$$B_{Lu} = \frac{\Delta B_L}{q} = \frac{\Delta B_L}{p \tan\theta} = \frac{b_L\ L(\sqrt{1+\tan^2\theta} - 1)}{\sqrt{3}\,V\,J\,\tan\theta} \qquad (7)$$

That is, about VARs transmitted throw transmission line .

Now, about the VARs transmission increases the apparent power and hence the rating of transformers. The transformer rating of one transformer substation $S_{T1}$ is:

$$S_{T1} = \sqrt{p^2 + q^2} = p\sqrt{1 + \tan^2\theta} \qquad (8)$$

$$S_{T2} = \frac{p\sqrt{1+\tan^2\theta}}{1.4} \qquad (9)$$

For a two – transformer substation $S_{T2}$ , the rating of each transformer is approximately sixty percent of the total load ( i.e., S/1.4) [18], [19].

The total cost of transformer for a one-transformer substation due to the added losses is:

$B_T$ = initial cost + add cost by increasing reactive power
= initial cost + add cost for VA * no. of VA
i.e.,

$$B_T = b_{OT} + b_T S = b_{OT} + b_T\,p\sqrt{1+\tan^2\theta} \qquad (10)$$

Where :
$b_{OT}$: initial cost of transformer in $ .
$b_T$ : additional cost per additional VA .
S : value of increasing in VA .

The equation of total cost without transmission reactive power (i.e., transmission purely active power) is:

$$B_{T2} = b_{OT} + b_T\,p\sqrt{1 + \tan^2 0} \qquad (11)$$

The additional cost due to the transmission of apparent power as compared with the transmission of purely active power is expressed as:

$$\Delta B_L = b_T\,p\left(\sqrt{1+\tan^2\theta} - 1\right) \qquad (12)$$

Therefore, the rise in cost per unit of reactive power transmission is, for a one-transformer substation:

$$B_{Tu1} = \frac{\Delta B_T}{q} = \frac{\Delta B_T}{P\tan\theta} = \frac{b_T\,P\left(\sqrt{1+\tan^2\theta} - 1\right)}{P\tan\theta}$$

$$= \frac{b_T(\sqrt{1+\tan^2\theta} - 1)}{\tan\theta} \qquad (13)$$

and for a two transformer substation:

$$B_{Tu2} = \frac{\Delta B_T}{p\tan\theta} = \frac{b_T(\sqrt{1+\tan^2\theta} - 1)}{1.4\tan\theta} \qquad (14)$$

## 7. ACTIVE POWER LOSS

Active power loss in the line is:

$$\Delta P_L = I^2 R_L = \frac{p^2 + q^2}{V^2} R_L = \frac{p^2}{V^2}(1 + \tan^2\theta) R_L \qquad (15)$$

For a balanced active power in the system, the generated output at the power station should be increased to meet the extra active power loss due to the transmission of VARS. Such an increase in the generated output is considered economically permissible if the cost due to the additional power loss does not exceed the cost of installing and maintaining the compensating VARs equipment at the consuming end [20], [21], i.e.,

$$K_a \Delta P_L \le K_r Q_r \qquad (16)$$

where:
$K_a$ = Cost / kW of generated output, $ / kW,
$K_r$ = Cost / kVAr of VARs compensation equipment , $ / kVAr,
$Q_r$ = kVAr rating of reactive power equipment , kVAr.

## 8. REACTIVE POWER LOSS

Reactive power transmission leads to voltage drop in transmission lines, The reactive power loss is:

$$\Delta Q_L = I^2 X_0 L = \frac{p^2}{V^2}(1 + \tan^2\theta) X_0 L \qquad (17)$$

where, $X_0$ is unit reactance of the line, Ω / km.

This loss is taken into account in the reactive power balance in the system. As such, the installed VARS source in the system should be increased to compensate for the loss.

**Note that**

VARS transmission from the generator has technical constraints.

VARS transmission is considered economical if the cost of generation at the power station, (including losses in the system) is less than or equal to the cost ( excluding losses in the system ) of installing VARS compensating equipment at the consuming end [20], [22], i.e.,

$$K_Q\left(Q_{beg} + \Delta Q_L\right) \le K_r Q_r \qquad (18)$$

Where:

$K_Q$ : cost energy loss due to VARS transmission,$/kWh,

$Q_{beg}$ : VARS output at the beginning of the line, kVAr.

Equations (17) and (18) are used to establish approximately the economic justifiable distance for VARS transmission by putting $Q_{beg} = Q_r$ (considering only power loss) as

$$K_Q(Q_r + \Delta Q_L) = K_r Q_r$$

$$K_Q \Delta Q_L = K_r Q_r - K_Q Q_r$$

$$\Delta Q_L = \frac{(K_r - K_Q)Q_r}{K_Q}$$

Since, $Q_r$ is expressed as $q = p\tan\theta$, then:

$$\Delta Q_L = \frac{(K_r - K_Q)p\tan\theta}{K_Q}$$

By putting this equation in equation (17), we get:

$$\frac{(K_r - K_Q)p\tan\theta}{K_Q} = \frac{p^2}{V^2}(1+\tan^2\theta)X_0 L$$

$$L = \frac{(K_r - K_Q)V^2 p\tan\theta}{K_Q X_0 p^2(1+\tan^2\theta)} \approx \frac{(K_r - K_Q)V^2 p\tan\theta}{K_Q X_0 p^2 \tan^2\theta}$$

$$L_r = \frac{(K_r - K_Q)V^2}{K_Q X_0 p\tan\theta} \qquad (19)$$

:

## 9. ENERGY LOSS

Energy loss is expressed as:

$$\Delta A_Q = \Delta P_Q \xi_Q \qquad (20)$$

where, $\xi_Q$ is average time / year, corresponding to the total time for VARS transmission, hr.

The energy loss leads to an increase in the use of fuel at the generating station. This increase in operational cost is obtained as [18], [20], [22]:.

$$C_F = \beta\sigma\Delta A_Q = \beta\sigma\frac{p^2}{V^2}\tan^2\theta\, R_L\, \xi_Q \qquad (21)$$

where;

$\beta$ : cost of fuel , $/m³ .

$\sigma$ : cubic metre of extra fuel used due to the compensation for the transmission of reactive power.

The cross-sectional area of the conductor is( from equation 2 ):

$$F = \frac{p\sqrt{1+\tan^2\theta}}{\sqrt{3}\,VJ} = \frac{p\sqrt{1+\frac{\sin^2\theta}{\cos^2\theta}}}{\sqrt{3}\,V\,J} = \frac{p\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}}}{\sqrt{3}\,V\,J}$$

$$= \frac{p\sqrt{\frac{1}{\cos^2\theta}}}{\sqrt{3}\,V\,J} = \frac{p}{\sqrt{3}\,V\,J\cos\theta} \qquad (22)$$

also,

$$F = \rho\frac{L}{R_L} \qquad (23)$$

From equation (22) and equation (23) we get:

$$R_L = \frac{L\rho\sqrt{3}\,V\,J\cos\theta}{p} \qquad (24)$$

Where, $\rho$ = resistivity of conductor, Ω mm² / km.

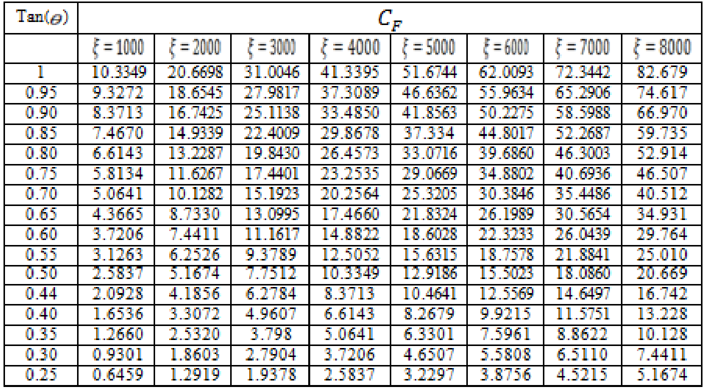
Hence,

$$C_F = \beta\sigma\frac{p^2}{V^2}\tan^2\theta\,\frac{L\rho\sqrt{3}\,V\,J\cos\theta}{p}\,\xi_Q$$

$$= \beta\sigma\frac{p}{V}\tan^2\theta\, L\rho\sqrt{3}\,J\cos\theta\,\xi_Q \qquad (25)$$

Now, we calculate $C_F$ ( increase in operational cost ) for transmission line from Haditha Dam substation to Qa'im substation in West of Iraq, for conductor of type Lark with (1) 132 KV single circuit overhead line have cross section(F) =248mm², V=132kV, $\beta\sigma$=0.001$ , $\rho$=0.0365 Ωmm²/km, L=1km and p=35000MW with different value of θ and $\xi_Q$. The MATLAB computer results in Table (1) and Figure (1) are obtained on the basis of these data.

**Table (1): The value of $C_F$ with different value of tan(θ) & $\xi_Q$**



| Tan($\theta$) | $C_F$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\xi = 1000$ | $\xi = 2000$ | $\xi = 3000$ | $\xi = 4000$ | $\xi = 5000$ | $\xi = 6000$ | $\xi = 7000$ | $\xi = 8000$ |
| 1 | 10.3349 | 20.6698 | 31.0046 | 41.3395 | 51.6744 | 62.0093 | 72.3442 | 82.679 |
| 0.95 | 9.3272 | 18.6545 | 27.9817 | 37.3089 | 46.6362 | 55.9634 | 65.2906 | 74.617 |
| 0.90 | 8.3713 | 16.7425 | 25.1138 | 33.4850 | 41.8563 | 50.2275 | 58.5988 | 66.970 |
| 0.85 | 7.4670 | 14.9339 | 22.4009 | 29.8678 | 37.334 | 44.8017 | 52.2687 | 59.735 |
| 0.80 | 6.6143 | 13.2287 | 19.8430 | 26.4573 | 33.0716 | 39.6860 | 46.3003 | 52.914 |
| 0.75 | 5.8134 | 11.6267 | 17.4401 | 23.2535 | 29.0669 | 34.8802 | 40.6936 | 46.507 |
| 0.70 | 5.0641 | 10.1282 | 15.1923 | 20.2564 | 25.3205 | 30.3846 | 35.4486 | 40.512 |
| 0.65 | 4.3665 | 8.7330 | 13.0995 | 17.4660 | 21.8324 | 26.1989 | 30.5654 | 34.931 |
| 0.60 | 3.7206 | 7.4411 | 11.1617 | 14.8822 | 18.6028 | 22.3233 | 26.0439 | 29.764 |
| 0.55 | 3.1263 | 6.2526 | 9.3789 | 12.5052 | 15.6315 | 18.7578 | 21.8841 | 25.010 |
| 0.50 | 2.5837 | 5.1674 | 7.7512 | 10.3349 | 12.9186 | 15.5023 | 18.0860 | 20.669 |
| 0.44 | 2.0928 | 4.1856 | 6.2784 | 8.3713 | 10.4641 | 12.5569 | 14.6497 | 16.742 |
| 0.40 | 1.6536 | 3.3072 | 4.9607 | 6.6143 | 8.2679 | 9.9215 | 11.5751 | 13.228 |
| 0.35 | 1.2660 | 2.5320 | 3.798 | 5.0641 | 6.3301 | 7.5961 | 8.8622 | 10.128 |
| 0.30 | 0.9301 | 1.8603 | 2.7904 | 3.7206 | 4.6507 | 5.5808 | 6.5110 | 7.4411 |
| 0.25 | 0.6459 | 1.2919 | 1.9378 | 2.5837 | 3.2297 | 3.8756 | 4.5215 | 5.1674 |

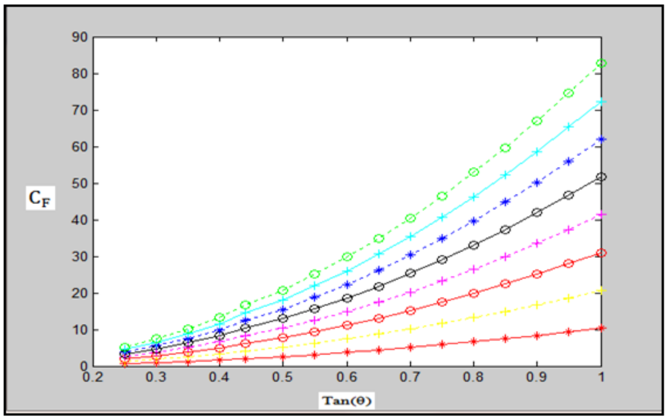


**Figure 1: The value of $C_F$ with different value of tan(θ) & $\xi_Q$**

It is clear that from Table (1) the changeability of average times which corresponding to the total time for VARS transmission ($\xi_Q$) for the same coefficient power ( tanθ ) are effectiveness in operational cost ($C_F$ ) of transmission line because the resistance and fuel used.

Now, systems cost is given by [22], [23]

$$C = \alpha K_r Q_r + C_F \qquad (26)$$

where, $\alpha$ = a depreciation factor of the VARS compensation equipment.

The first component of equation (26) reflects capital cost and the second component, operational cost.

From equations (26) and (25),

$$C = \alpha K_r Q_r + \beta\sigma\xi_Q \frac{P}{V}\tan^2\theta\rho L\sqrt{3}\,J\cos\theta \qquad (27)$$

The additional cost is minimum if ,

$$\frac{\partial C}{\partial tan\theta} = 0$$

Applying partial differentiation with respect to tan$\theta$

$$\frac{\partial C}{\partial \tan\theta} = \alpha\overline{K_r}\,P\sec^2\theta + \beta\sigma\xi_q\frac{P}{V}\rho Lj\sqrt{3}(-\tan^2\theta\sin\theta + 2\cos\theta\sec^2\theta\tan\theta)$$

multiplying by $V/psec^2\theta$

$$\frac{\partial C}{\partial \tan\theta} = 0 \Rightarrow \alpha\overline{K_r}V + \beta\sigma\xi_q\rho Lj\sqrt{3}(-\sin^3\theta + 2\sin\theta) = 0$$

The approximate distance for VARS transmission (considering minimum losses in the system) is obtained as:

$$-\alpha\overline{K_r}\,V = \beta\sigma\xi_q\rho Lj\sqrt{3}(2\sin\theta - \sin^3\theta)$$

$$L = \left|\frac{-\alpha\overline{K_r}V}{\beta\sigma\xi_q\rho j\sqrt{3}(2\sin\theta - \sin^3\theta)}\right|$$

$$L = \frac{\alpha\overline{K_r}V_{end}}{\beta\sigma\xi_q\rho j\sqrt{3}(2\sin\theta - \sin^3\theta)} \qquad (28)$$

where, $V_{beq}$ is Voltage at the end of the line, kV.

If the voltage drop, due to VARS transmission is considered then,

$$L = \frac{\alpha\overline{K_r}\,(V_{beg}V_{nom} - P\tan\theta\,XL)}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}$$

$$L = \frac{\alpha\overline{K_r}\,V_{beg}V_{nom} - \alpha\overline{K_r}\,P\tan\theta\,XL}{V_{nom}\beta\sigma\xi_q\,\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}$$

$$L\left(\frac{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta) + \alpha\overline{K_r}P\tan\theta\,X}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}\right)$$

$$= \frac{\alpha\overline{K_r}\,V_{beg}V_{nom}}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}$$

$$L = \frac{\alpha\overline{K_r}\,V_{beg}V_{nom}}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta) + \alpha\overline{K_r}\,P\tan\theta X} \qquad (29)$$

$$L + \frac{\alpha\overline{K_r}\,P\tan\theta\,X\,L}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)} = \frac{\alpha\overline{K_r}\,V_{beg}V_{nom}}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}$$

$$L(1 + \frac{\alpha\overline{K_r}\,P\tan\theta\,X}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}) = \frac{\alpha\overline{K_r}\,V_{beg}V_{nom}}{V_{nom}\beta\sigma\xi_q\rho\sqrt{3}\,j(2\sin\theta - \sin^3\theta)}$$

where:

$V_{beq}$ : voltage at the beginning of the line,

$V_{nom}$ : nominal voltage, kV.

The voltage at the end of the line in equation (26) can be expressed as,

$$V_{end} = V_{beg} - \frac{q\,X\,L}{V_{nom}} = V_{beg} - \frac{P\tan\theta\,XL}{V_{nom}}$$

$$= \frac{V_{beg}V_{nom} - p\tan\theta X_0 L}{V_{nom}} \qquad (30)$$

## 10 ECONOMICALLY JUSTIFIED DISTANCE FOR REACTIVE POWER TRANSMISSION

All the factors presented in previous sections are considered to establish the total cost, $C_T$ due to the transmission of VARS.

$$C_T = \alpha\left[\Delta B_L + K_a \Delta P + K_q\left(q_{beg} + \Delta q_L\right)\right] + C_F \qquad (31)$$

Substitution of appropriate values from equations (6), (15), (18) and (25) into equation (31) expands $C_T$ into,

$$C_T = \alpha\left[\frac{b_L PL}{\sqrt{3}Vj}\left(\sqrt{1+\tan^2\theta} - 1\right) + K_a \frac{P}{V}(1+\tan^2\theta)\rho L\sqrt{3}j\cos\theta + \right.$$

$$\left. K_q\left(P\tan\theta + \frac{P^2}{V^2}\tan^2\theta XL\right)\right] + \beta\sigma\xi_q \frac{P}{V}\tan^2\theta\, \rho L\sqrt{3}\, j\cos\theta \qquad (32)$$

For VARS transmission to be economically justified, $C_T$ must be minimum, i.e.

$$\frac{\partial C_T}{\partial \tan\theta} = \frac{\alpha b_L PL}{\sqrt{3}jV}\frac{\tan\theta}{\sqrt{1+\tan^2\theta}} + \alpha K_a \rho L\sqrt{3}j\frac{P}{V}\left(2\sin\theta - \sin^3\theta\right) +$$

$$\alpha K_q P + \alpha K_q \frac{P^2}{V^2} 2\tan\theta\ XL + \beta\sigma\xi_q \rho L\sqrt{3}\, j\frac{P}{V}(2\sin\theta - \sin^3\theta) = 0 \qquad (33)$$

The economically justified distance for VARS transmission is obtained from equation (33) as:

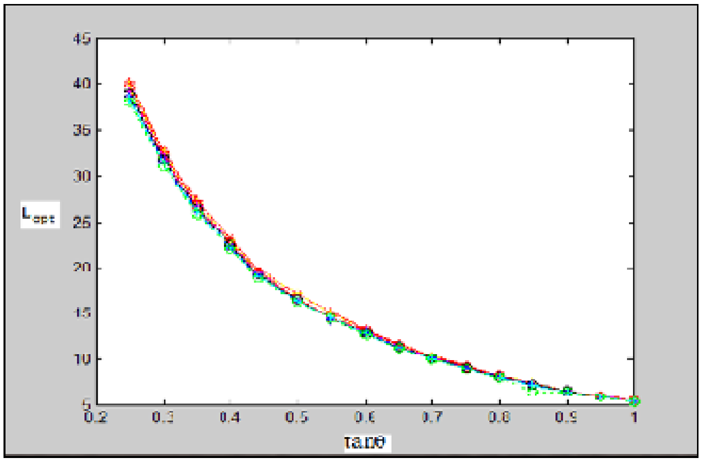
$$L_{opt} = \frac{\alpha K_q V^2}{\alpha\tan\theta\left(\frac{b_L V}{\sqrt{3}j\sqrt{1+\tan^2\theta}} + 2K_q PX\right) + \sqrt{3}\rho jV(2\sin\theta - \sin^3\theta)(\alpha K_a + \beta\sigma\xi_q)} \qquad (34)$$

Equation (34) is the mathematical model, which is used to compute the economically justified distance for VARS transmission.

Now, we calculate $L_{opt}$ for transmission line from Haditha Dam substation to Qa'im substation in West of Iraq, for conductor of type Lark with 132 KV single circuit overhead line have cross section(F) =248mm², V=132kV, $\beta\sigma$=0.001$ , $\rho$=0.0365 Ωmm²/km, L=1km and p=35000MW with different value of θ and $\xi_Q$. The MATLAB computer results in Table (2) and Figure (2) are obtained on the basis of these data.

Table(2): Economically justified distance for VARS transmission

| Tan(θ) | $L_{opt}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\xi = 1000$ | $\xi = 2000$ | $\xi = 3000$ | $\xi = 4000$ | $\xi = 5000$ | $\xi = 6000$ | $\xi = 7000$ | $\xi = 8000$ |
| 1 | 5.4379 | 5.4155 | 5.3924 | 5.3696 | 5.3470 | 5.3245 | 5.3015 | 5.2795 |
| 0.95 | 6.0037 | 5.9768 | 5.9502 | 5.9237 | 5.8976 | 5.8716 | 5.8460 | 5.8205 |
| 0.90 | 6.6334 | 6.6287 | 6.5977 | 6.5669 | 6.5365 | 6.5063 | 6.4505 | 6.4210 |
| 0.85 | 7.3856 | 7.3682 | 7.3320 | 7.2962 | 7.2607 | 7.2255 | 7.1723 | 6.4210 |
| 0.80 | 8.2435 | 8.2112 | 8.1689 | 8.1270 | 8.0856 | 8.0446 | 7.9939 | 7.9538 |
| 0.75 | 9.2279 | 9.1778 | 9.1283 | 9.0793 | 9.0309 | 8.9830 | 8.9358 | 8.8890 |
| 0.70 | 10.3638 | 10.3046 | 10.2463 | 10.1887 | 10.1316 | 10.0753 | 10.0199 | 9.9648 |
| 0.65 | 11.6858 | 11.6164 | 11.5508 | 11.4836 | 11.4171 | 11.3514 | 11.2813 | 11.216 |
| 0.60 | 13.2374 | 13.1553 | 13.0853 | 13.0073 | 12.9302 | 12.8540 | 12.7595 | 12.683 |
| 0.55 | 15.0797 | 14.9811 | 14.5527 | 14.4601 | 14.3687 | 14.2784 | 14.5150 | 14.424 |
| 0.50 | 17.1029 | 17.1761 | 16.6871 | 16.5763 | 16.4669 | 16.3590 | 16.4351 | 16.328 |
| 0.44 | 19.8149 | 19.6670 | 19.5307 | 19.3963 | 19.2637 | 19.1329 | 19.0317 | 18.907 |
| 0.40 | 23.1296 | 22.9144 | 22.7505 | 22.589 | 22.4297 | 22.2727 | 22.1649 | 22.011 |
| 0.35 | 27.2428 | 27.0398 | 26.8398 | 26.6427 | 26.4485 | 26.2572 | 26.0685 | 25.882 |
| 0.30 | 32.6961 | 32.4456 | 32.1988 | 31.9558 | 31.7164 | 31.4806 | 31.2483 | 31.019 |
| 0.25 | 40.1974 | 39.8814 | 39.5704 | 39.2642 | 38.9626 | 38.6657 | 38.3733 | 38.085 |



Figure(2): Graph for VARS trransmission

It is clear that from Table (2) the changeability of average times which corresponding to the total time for VARS transmission ($\xi_Q$) for the same coefficient power ( tanθ ) are not effectiveness in transmission line.

## 6. CONCLUSION

The cost of electrical equipment is high and continues to rise yearly. It is expected of power systems engineers to design systems that are not only reliable and stable, but also can operate economically.

This paper has established a mathematical model that could be used to help determine the economically justified distance for VARS transmission. The computer results are satisfactory and could be of guidance to consulting and power systems design engineers who have to justify their projects technically and economically, especially in choosing conductor and equipment sizes / ratings.

## REFERENCES

[1] Tokad, Y., 1985, "Analysis of Engineering Systems-Part 2" (in Turkish ), YI1dtz University Press, Istanbul.

[2] Unal, A., 1986, "Determination and Solution of State-Equations -Solved Problems" Couree Notes (in Turkish), Yl1d1zUniversity Press, Istanbul.

[3] Ünal, A., 1996, Determination of mathematical model of an electric power system using linear graph, Mathematical & Computational Applications, Vol. 1, No.1, pp: 134-139.

[4] Anderson, G. O., and Ogwu, F. J., 2002, "Development of a mathematical model for VARs transmission in electric power systems", IEEE Africon, Vol. 2, pp: 941- 945.

[5] Mehta, V.K and Mehta, R., 2008, Principles of Power Systems, S. Chand & Company Ltd, New Delhi.

[6] Gupta, J. B., 2008, A Course in Power System, S.K. Kataria & Sons, Publisher of Engineering and Computer books, New Delhi.

[7] Atandare, D.L. Nigerian's Epileptic power supply - the way out; Prof. E.K. Obiakor Lecture Series 8, The Federal Polytechnic, Ado-Ekiti, 2007.

[8] Wadhwa, C.L., 2009, Electrical Power Systems, New Age International (P) Limited, Publishers, New Delhi.

[9] Bamigbola, O.M. and Aderinto, Y.O., 2009, On the Characterization of Optimal Control Model of Electric power Generating Systems, International Journal of Physical Sciences, Vol. 4, No.1.

[10] Aderinto, Y.O., 2010, An Optimal Control Model of the Electric Power Generating System, Unpublished Ph.D. Thesis, Department of Mathematics, University of Ilorin, Nigeria.

[11] Okafor, C.E. and Adebanji, B., 2009, An Assessment of Transmission System Reliability in Nigeria, Journal of Research in Engineering (JRENG), Vol. 6, pp: 21-34.

[12] Bagriyanik, F. G., Aygne, Z. E. and Bagriyanik, M., 2003, Power Loss Minimization Using Fuzzy Multi-objective Formulation and Genetic Algorithm, Presented at IEEE Power Tech Conference, June 23rd 26th, Bologna, Italy.

[13] Onohaebi, O.S. and Odiase, O.F., 2010, Empirical Modeling of Power Losses as a Function of Line Loadings and Lengths in the Nigerian 330KV Transmission Lines, International Journal of Academic Research, pp: 47-53.

[14] Oke, M. O., July - Aug 2012, Mathematical Model for the Determination of Voltage and Current on Lossy Power Transmission Lines IOSR Journal of Mathematics (IOSRJM), ISSN: 2278-5728 Vol. 1, Issue 4, PP: 16-18.

[15] Kueck, J., Kirby, B., Rizy, T., Li, F., and Fall, N., 2006, Reactive Power from Distributed Energy, The Electricity Journal, Vol.19, Issue10 , PP: 26 - 38.

[16] Sauer, P. W., 2003, reactive power and voltage control issues in electric power system , applied mathematics for power system,pp:11– 24.

[17] Manlike, N. A., 1983, Reactive Power Compensation in Power Systems, Energy Journal, Moscow.

[18] Aberson, M. I., 1989, Compensation of Reactive Power in Modern Power Systems, IEEE Journal, London.

[19] Vijay, B., Bodger, P., and Wood, A., September 8-10, 2008, Towards a Practical Partial Core Transformer- Compensation of Reactive Power Requirements with a VSC, Proceedings of the Second IASTED Africa Conference on Power and Energy Systems, Gaborone, Botswana, pp: 32-37.

[20] Neklepayev, B. N., 1982, Elecric Stations and Systems Reference Book, High School Publishers, Moscow.

[21] Oyedokun, D. T., Folly, K. A., September 8-10, 2008, Power Flow Studies in HVAC and HVDC Transmission Lines, Proceedings of the Second IASTED Africa Conference on Power and Energy Systems.

[22] Kasak, N. A., 1985, Technico – Economic Calculations of Reactive Power Compensation in Networks, Electrichestvo Journal, Moscow.

[23] galu, Y., Munda, L., Jimoh, A. A., September 8-10, 2008, Sability Analysis of Centurion Electric Power System, Proceedings of the Second Iasted Africa Conference on Power and Energy Systems, Gaborone, Botswana, pp: 51- 56.